Transmission of linearly polarized light through a single-mode fiber with random fluctuations of birefringence

Gregory D. VanWiggeren and Rajarshi Roy

A simple theoretical formalism is developed to describe the effect of transmission on linearly polarized light through a fiber with random fluctuations of birefringence. We conclude that, for any optical fiber that does not experience polarization-dependent gain or loss, there exist two orientations for linearly polarized light input into the optical fiber that will also exit the fiber linearly polarized. We report experimental results that verify this prediction and also investigate its practical implications and limitations; in particular we investigate the stability of these linearly polarized output states in laboratory conditions. © 1999 Optical Society of America

OCIS codes: 060.2310, 060.2400, 060.2430, 060.2420, 260.1440, 260.5430.

1. Introduction

When one works with fiber-optic systems in the laboratory, it is often desirable to couple linearly polarized light into a fiber and to obtain linearly polarized light at the output as well. One way to accomplish this is to use polarization-maintaining fibers. The high birefringence of these fibers allows for linearly polarized light launched along the proper axis to travel long distances without change of polarization state. In ordinary single-mode fiber, however, the polarization state evolves rapidly as the light propagates. The output polarization will appear uncorrelated to the input polarization after only a few meters of propagation. However, it can be shown that unperturbed single-mode fiber can perform the same function as polarization-maintaining fiber in certain situations. In this paper we predict theoretically and demonstrate experimentally that, for any ordinary single-mode optical fiber, two orientations of linearly polarized quasi-monochromatic input light will also exit the fiber linearly polarized. When the ordinary single-mode fiber is not perturbed by external stresses or temperature changes, these states are reasonably stable for hours—especially for shorter fiber lengths (100 m or less).

Earlier researchers[1–4] have also observed linearly polarized input and output of light through a fiber. In Refs. 1 and 2 it is noted that linear light input into a fiber can be output linearly polarized as well, but no theoretical explanation is provided. In Refs. 3 and 4 the observations of linearly polarized input and output of light through a fiber were explained with a model that treats the birefringence of the optical fiber as constant in magnitude and orientation throughout the length of the fiber. This assumption is not, in general, correct. Much of the research into the propagation of polarized light in optical fiber has been in the context of polarization-mode dispersion,[5,6] a subject that is not addressed in this paper.

Here the birefringence fluctuations along a fiber are treated as a concatenation of wave plates with each wave plate possessing an arbitrary birefringence. The simple Jones matrix formalism used to analyze such a concatenation provides a framework for understanding many polarization phenomena observed in optical fiber, including polarization-mode dispersion[5] and four-wave mixing in single-mode fiber.[7] The formalism can be used to show that the operation of randomly fluctuating birefringences in an optical fiber is the same as the operation of only one constant birefringence for the whole fiber, as was assumed in Refs. 3 and 4. The formalism provides a simple way of mathematically determining the particular orientation of linearly polarized input light that will also exit a fiber linearly polarized. It

---

The authors are with the School of Physics, Georgia Institute of Technology, Atlanta, Georgia 30332. The e-mail address for G. D. VanWiggeren is greg@socrates.physics.gatech.edu.

Received 23 October 1998; revised manuscript received 3 March 1999.

0003-6935/99/183888-05$15.00/0

clearly indicates that only two such orientations can exist, that they are orthogonal, and that in general they are not eigenpolarizations. In fact, this formalism shows that the polarization state of eigenpolarized light can evolve as it propagates through the fiber. Finally, the framework offers a way of understanding the effects of polarization-dependent gain or loss in an optical fiber. Experiments verifying these predictions and demonstrating their usefulness in a laboratory setting are performed and the results given in Sections 3 and 4.

2. Theory

At any point along a single-mode fiber the local birefringence is typically of the order of $10^{-7} < (n_1 - n_2)/\sqrt{n_1 n_2} < 10^{-5}$.[8] Even such a small birefringence can lead to large changes in the polarization state of light over 1 m. In real fibers the magnitude of the birefringence is never constant throughout a length of fiber. Instead, it fluctuates according to whatever local stresses, internal or external, exist in the fiber. To complicate matters further, the orientation of the index ellipsoid rotates unpredictably from one point to the next in fiber and is sensitive to movement of the fiber or to changes in temperature. The birefringence in an optical fiber is also a function of wavelength, but this wavelength dependence is not accounted for in the analysis given. Consequently, the results from the analysis presented in this paper are valid only in situations in which this wavelength dependence can be neglected. This condition is often well satisfied in laboratory settings where the bandwidths of the optical sources are typically 1 nm or less and the lengths of fiber are less than a few kilometers.

As mentioned above, a length of single-mode optical fiber is modeled here as a concatenation of differential elements, each element a wave plate possessing a birefringence of arbitrary orientation and magnitude. After passing through one element, the light passes into a second element, and so on, until it reaches the end of the fiber. In a Jones matrix representation[9] this process can be described a series of phase-shift and rotation matrices[10,11]:

A phase shift of δ is described by the matrix

$$C(\delta) = \begin{bmatrix} \exp(i\delta/2) & 0 \\ 0 & \exp(-i\delta/2) \end{bmatrix}. \tag{1}$$

In the model the phase shift is given by $\delta = 2\pi L(n_1 - n_2)/\lambda$, where L is the length of the differential element and n_1 and n_2 are the indices of refraction along the fast and slow axes, respectively. A rotation of the index ellipsoid by an angle θ is represented by the matrix

$$\mathbf{R}(\theta) = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}. \tag{2}$$

The polarization properties of a length of fiber, then, can be represented as a product of many unknown phase-shift matrices and rotation matrices:

$$\mathbf{M} = \mathbf{C}(\delta)\mathbf{R}(\theta)\mathbf{C}(\epsilon)\mathbf{R}(\phi)\ldots. \tag{3}$$

The product of a phase-shift matrix and a rotation matrix can produce any arbitrary unitary matrix. Consequently, the form of **M** is also the most general form of a unitary matrix:

$$\mathbf{M} = \begin{bmatrix} a & b \\ -b^* & a^* \end{bmatrix}, \tag{4}$$

where $|a^2| + |b^2| = 1$. The unitary nature of the matrix **M** allows for it to be decomposed into only one appropriate phase-shift matrix **C** multiplied by one rotation matrix **R**. Interestingly, this suggests that any fiber's net effect on the polarization state of light is identical to the effect of one particular wave plate with a constant phase shift and orientation of its axes.

A proof for the form of **M** can be given quickly[11] with the Pauli matrix

$$\Sigma = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}.$$

Now Σ has the following properties:

$$\Sigma^2 = -I,$$

where I is the identity matrix,

$$\Sigma\mathbf{R}(\theta)\Sigma = -\mathbf{R}(\theta), \tag{5}$$

$$\Sigma\mathbf{C}\Sigma = -\bar{\mathbf{C}},$$

where the elements of $\bar{\mathbf{C}}$ are the complex conjugates of the elements of **C**. If a general matrix form for **M** is assumed, then

$$\mathbf{M} = \begin{bmatrix} a & b \\ c & d \end{bmatrix},$$

where the elements are all complex. Performing the matrix multiplication gives

$$\Sigma\mathbf{M}\Sigma = -\begin{bmatrix} d & -c \\ -b & a \end{bmatrix}. \tag{6}$$

From Eq. (3) it is also true that

$$\begin{aligned} \Sigma\mathbf{M}\Sigma &= \Sigma\mathbf{C}(\delta)\mathbf{R}(\theta)\mathbf{C}(\epsilon)\mathbf{R}(\phi)\ldots\mathbf{C}(\gamma)\mathbf{R}(\psi)\Sigma \\ &= -\Sigma\mathbf{C}(\delta)\Sigma^2\mathbf{R}(\theta)\Sigma^2\ldots\Sigma^2\mathbf{C}(\gamma)\Sigma^2\mathbf{R}(\psi)\Sigma \\ &= -\bar{\mathbf{C}}(\delta)\mathbf{R}(\theta)\bar{\mathbf{C}}(\epsilon)\mathbf{R}(\phi)\ldots\bar{\mathbf{C}}(\gamma)\mathbf{R}(\psi) \\ &= -\bar{\mathbf{C}}(\delta)\bar{\mathbf{R}}(\theta)\bar{\mathbf{C}}(\epsilon)\bar{\mathbf{R}}(\phi)\ldots\bar{\mathbf{C}}(\gamma)\bar{\mathbf{R}}(\psi) \\ &= -\bar{\mathbf{M}} = -\begin{bmatrix} a^* & b^* \\ c^* & d^* \end{bmatrix}. \end{aligned} \tag{7}$$

When we take the results from Eqs. (6) and (7), it is clear that $d = a^*$ and $c = -b^*$ and that the form of **M** given in Eq. (4) has been proved.

Light in a fiber has electric-field components along two orthogonal transverse axes. These axes are chosen arbitrarily and denoted here as x and y. In the Jones matrix representation these complex components are represented as a vector $\mathbf{j}_{\text{in}} = (E_x, E_y)$. The output fields are represented by $\mathbf{j}_{\text{out}} = (E_x', E_y')$. The transformation of light as it propagates through the length of the fiber is thus given by

$$\mathbf{j}_{\text{out}} = \mathbf{M} \cdot \mathbf{j}_{\text{in}}. \tag{8}$$

To obtain linearly polarized output from the fiber, the ratio E_x'/E_y' must be real, though both components are in general complex. In other words, the input electric-field components E_x and E_y must satisfy the condition

$$\text{Im}\left(\frac{E_x'}{E_y'}\right) = 0 = \frac{aE_x + bE_y}{-b^*E_x + a^*E_y} - \text{c.c.}, \tag{9}$$

where c.c. is the complex conjugate.

The ratio of the input electric-field components E_x/E_y is also real, because the input light is linearly polarized. Rewriting Eq. (9) in terms of the ratio $r = E_x/E_y$ and rationalizing gives

$$r^2 - \frac{\text{Im}(a^2 - b^2)}{\text{Im}(ab)} r - 1 = 0. \tag{10}$$

The two solutions for r given by Eq. (10) correspond to orientations of linearly polarized input light that will also exit the fiber linearly polarized. By using these solutions for r to construct the vectors $\mathbf{j}_{\text{in}+}$ and $\mathbf{j}_{\text{in}-}$ and then taking the dot product, we can show that the two solutions, and thus their corresponding input orientations, are orthogonal. Finally, solving Eq. (8) for $\mathbf{j}_{\text{out}+}$ and $\mathbf{j}_{\text{out}-}$, we can show that the output orientations are also orthogonal. It should be noted that the polarization states of these solutions are not maintained as they propagate; instead, they evolve continuously. The evolution of these particular orientations, however, is precisely such that the light exits the fiber linearly polarized. It is also important to note that neither $\mathbf{j}_{\text{in}+}$ nor $\mathbf{j}_{\text{in}-}$ is an eigenvalue. Although both the input and the output are linearly polarized, the angular orientation is not necessarily the same.

A less idealized model for a length of optical fiber would have to include effects such as gain or loss. For long fibers, loss may be significant. Gain and loss can be incorporated into the description given above by simple inclusion of an appropriate Jones matrix for a polarization-dependent gain or loss,

$$\mathbf{G}(c, d) = \begin{bmatrix} c & 0 \\ 0 & d \end{bmatrix}, \tag{11}$$

where c and d are real.

The gain or loss need not be oriented along the same direction as the birefringence, but an extra rotation matrix in each differential element would compensate for this. Thus

$$\mathbf{M} = \mathbf{C}(\delta)\mathbf{R}(\theta)\mathbf{G}(c, d)\mathbf{R}(\phi)\mathbf{C}(\epsilon)\mathbf{R}(\psi)\mathbf{G}(e, f)\mathbf{R}(\zeta)\ldots. \tag{12}$$

By applying $\mathbf{G}$ to the framework developed above, we can show that if $c = d$ then the effect of such a loss or gain is simply to multiply $\mathbf{M}$ by a constant value. In this case the discussion above is unaffected. However, if the elements of $\mathbf{G}$ are not identical, $c \neq d$, in each differential element, the proof given above no longer holds. Stated another way, if a fiber has polarization-dependent gain or loss, $\mathbf{M}$ will not have the same form as in Eq. (4), and the treatment given above will no longer apply.

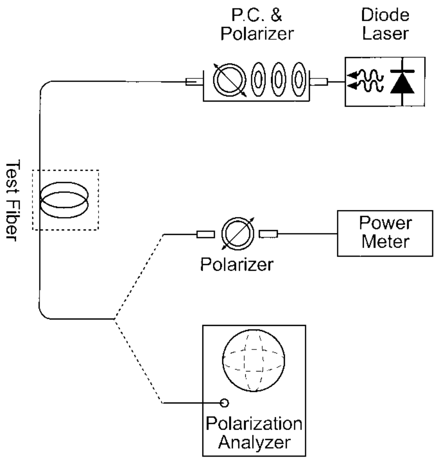

Fig. 1. Experimental setup for measuring polarization of a test fiber. The extinction ratio results were obtained with the second polarizer and the power meter. The results shown in Figs. 2–4 were obtained with the polarization analyzer after the test fiber. P.C., polarization controller.

3. Experiment

Birefringences in a fiber, and thus the matrix $\mathbf{M}$ for that fiber, are highly sensitive to movement and to other environmental perturbations. It is important, therefore, to verify that the theory's predictions can be demonstrated experimentally and to investigate the conditions in which the theory applies.

Figure 1 shows the basic setup. A tunable diode laser is used to produce 4 mW of light with a wavelength of 1550 nm and a 150-kHz linewidth. The light propagates down a fiber and is coupled to free space with a graded-index (GRIN) lens where it passes through a sequence of three wave plates ($\lambda/2$, $\lambda/4$, $\lambda/2$). These three wave plates operate on the light from the tunable diode laser to ensure that the light is roughly circularly polarized when incident on the polarizer so that roughly equal power is transmitted through the polarizer as it is rotated. Another GRIN lens couples the light that passes through the polarizer into the test fiber.

In the first set of experiments the light transmitted by the first polarizer is launched into the test fiber

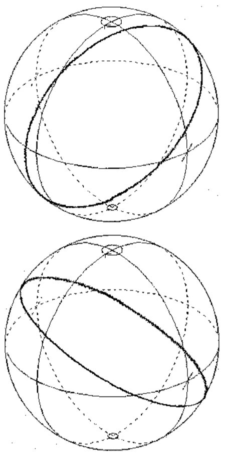
(a) (b)

Fig. 2. Polarization state paths (thicker curves) traced at the output of the fiber as the polarizer at the input is rotated through 180°. (a) 8-m test fiber. (b) 35-km test fiber.

and propagates until it reaches a second polarizer. The light that passes through the second polarizer is then input into an optical power meter. The two polarizers are adjusted to achieve maximum extinction as measured by the power meter. Experimentally, two orientations of the first polarizer were found, which allowed for maximum extinction. Thus the experiment demonstrates that two orientations of linearly polarized input light gave linearly polarized light at the output of the test fiber. This was true for both the short, 8-m, and the long, 35-km, lengths of fiber that were tested. For both lengths it was also observed that the orientations of the two linearly polarized inputs were orthogonal to within a measurement error of ±0.5°. The linearly polarized output, to within the same error, was also orthogonal. All of this is as predicted by the proof developed above. The extinction ratio for the short test fiber was >45 dB, whereas for the 35-km sample it had fallen to 35 dB. In all cases the ratio is large enough to accurately determine the location of the extinction maxima. The decrease in the extinction ratio is most probably due to scattering phenomena that depolarize the propagating light.

As a measure of the wavelength dependence of this result, amplified spontaneous emission light from an erbium-doped fiber amplifier (EDFA) was used as the source. The amplified spontaneous emission light is very broadband, possessing a 3-dB bandwidth of approximately 5 nm centered at 1532 nm. Even with such broadband light, a >15-dB extinction ratio was obtained after propagation through an approximately 1.5-km optical fiber.

Another set of experiments was performed with a polarization analyzer. For the 8-m and the 35-km test fibers mentioned above, a series of measurements were made in which the input polarizer was rotated, and the resulting polarization states were tracked on the Poincaré sphere. As can be seen in Fig. 2, rotating the polarizer through 180° causes great-circle paths to be traced out on the Poincaré sphere. The paths intersect the equator twice, and on opposite sides of the sphere. Because the equator of a Poincaré sphere represents linear polarization ($s_3 = 0$), these paths indicate that two orientations of the input polarizer will result in linearly polarized light output from the fiber. The fact that the intersections with the equator occur on opposite sides of the sphere demonstrates that the two linearly polarized output states are orthogonal, as predicted by the theory. The different paths traced around the Poincaré sphere shown in Fig. 2 for the same fiber are the result of simple rearrangement of the way the fiber lay on the experimental table. This is simply a manifestation of the fact that the polarization properties of fiber are sensitive to changes in external stresses.

For single-mode fiber to be useful in transmitting linearly polarized light from one place to another, the polarization state of the output light should be relatively stable under laboratory conditions. The polarization state of light output from a fiber was tracked over time by a polarization analyzer. As seen in Fig. 2, the polarization state of the output light is represented as a point on the Poincaré sphere. Figure 3 shows the wandering of that point with time. In Fig. 3(a), the 8-m case, the fiber was coiled and lay on an optical table. The data shown were obtained over the course of 32 h. The relatively small excursions show that the polarization state was fairly sta-

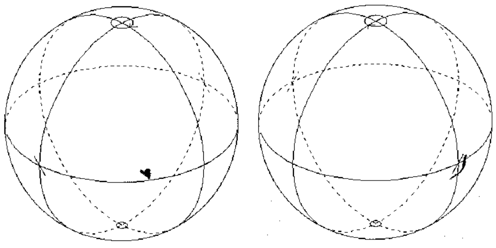
(a) (b)

Fig. 3. Smear of points on the Poincaré sphere represent the evolution of the output polarization from an 8-m length of optical fiber during 32 h under typical laboratory conditions. (b) Same experiment showing the evolution of the output state of polarization of a 35-km length of fiber over only 4 h. Both experiments are intended to give some practical indication of the stability of the output polarization of a single-mode fiber when the input polarization is held constant.

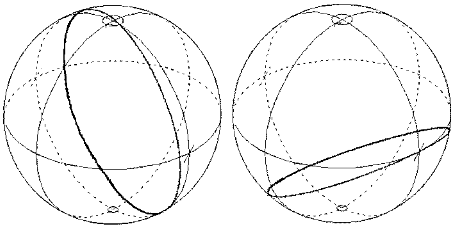
(a) (b)

Fig. 4. (a) Polarization path traced by output light after experience of a small polarization-dependent gain in an EDFA. (b) Path traced by output light experiencing polarization-dependent loss.

ble over that entire time. Figure 3(b) was obtained over a period of 4 h with a 35-km length of fiber wrapped around a spool in the same laboratory setting. Not surprisingly, the length of the fiber seems to have an effect on the stability of the polarization state over time.

4. Effects of Loss and Gain

Two more experiments were performed to test the effect of polarization-dependent loss and gain on the polarization properties of a fiber. In the first experiment an EDFA was used as a source of polarization-dependent gain. The amplifier's specifications suggest a polarization-dependent gain of <0.3 dB, which is consistent with the very small (difficult to measure) ~ 0.1 dB of polarization-dependent gain observed in the laboratory for a small-signal gain of ~ 27 dB. Although the EDFA gives an almost polarization-independent amplification of signals, it seems plausible that the winding of the doped fiber and the polarization of the pump lasers would contribute to polarization-dependent gain within individual elements along the fiber. This would result in local differential element gain matrices, $\mathbf{G}(c, d)$, which have $c \neq d$. Although the total output power might be almost polarization independent, the form of $\mathbf{M}$ would be different from Eq. (4). This polarization gain dependence results in the deviation from a great circle that is observed in Fig. 3(a).

Another experiment was conducted to investigate the effect of polarization-dependent loss. In the test fiber portion of the experiment the light was coupled to free space, again with a GRIN lens. From there it passed through a birefringent calcite crystal that acts as a polarizing beam splitter. When we controlled the angle of a glass plate behind the crystal, the amount of light from each polarization that was coupled through a GRIN lens and back into the test fiber could be controlled. Depending on the angle of the glass plate, one polarization or the other could be coupled preferentially back into the test fiber. This device acts as a polarization-dependent loss. When such a polarization-dependent loss is created, the path on the Poincaré sphere that results from rotation of the input polarizer is not a great circle, as is evident in Fig. 3(b).

5. Conclusion

Using the Jones matrix formalism, we have proved that two orthogonal orientations of linearly polarized light can be launched into any single-mode fiber such that linearly polarized light is output from the fiber. Experiments were performed that verified these predictions. They also revealed, in accordance with the model, that these same predictions cannot be extended to the case of fiber with polarization-dependent gain or loss.

Knowing that ordinary single-mode fiber can transmit linearly polarized light may have some practical applications in laboratory settings. As with polarization-maintaining fiber, the proper axes for input and output must be found before this property can be used. Unlike with polarization-maintaining fiber, when we obtain linearly polarized input and output light with ordinary single-mode fiber, precautions must be taken against perturbing the fiber once these axes have been found. Although the particular orientation of linearly polarized input and output light for an unperturbed length of fiber may drift on a time scale of minutes or hours, it seems plausible that a simple feedback algorithm could be developed for maintaining the proper axes indefinitely.

We acknowledge support from the Office of Naval Research. We thank P. O'Shea, J. Buck, and A. Judy for helpful discussions and C. Verber for essential instrumentation.

References

1. S. C. Rashleigh, W. K. Burns, R. P. Moeller, and R. Ulrich, "Polarization holding in birefringent single-mode fibers," Opt. Lett. **7,** 40–42 (1981).
2. H. G. Winful and A. Hu, "Intensity discrimination with twisted birefringent optical fibers," Opt. Lett. **11,** 668–670 (1986).
3. V. Ramaswamy, R. Standley, D. Sze, and W. G. French, "Polarization effects in short length single mode fibers," Bell Syst. Tech. J. **57,** 635–651 (1978).
4. K. Okamoto, Y. Sasaki, T. Miya, M. Kawachi, and T. Edahiro, "Polarisation characteristics in long length V.A.D. single-mode fibres," Electron. Lett. **16,** 768–769 (1980).
5. B. L. Heffner, "PMD measurement techniques—a consistent comparison," in *Optical Fiber Communication Conference*, Vol. 2 of 1996 OSA Technical Digest Series (Optical Society of America, Washington, D.C., 1996), pp. 292–293.
6. C. D. Poole and R. E. Wagner, "Phenomenological approach to polarisation dispersion in long single-mode fibres," Electron. Lett. **19,** 1029–1030 (1986).
7. K. Inoue, "Polarization effect on four-wave mixing efficiency in a single-mode fiber," IEEE J. Quantum. Electron. **28,** 883–894 (1992).
8. S. C. Rashleigh, "Origins and control of polarization effects in single-mode fibers," J. Lightwave Technol. **LT-1,** 312–331 (1983).
9. E. Hecht, *Optics*, 2nd ed. (Addison-Wesley, Reading, Mass., 1990), Chap. 8.
10. R. Roy, C. Bracikowski, and G. E. James, "Dynamics of a multimode laser with nonlinear birefringent intracavity elements," in *Recent Developments in Quantum Optics*, R. Inguva, ed. (Plenum, New York, 1993), pp. 309–328.
11. G. E. James, "Models of intracavity frequency-doubled lasers," Ph.D. dissertation (Georgia Institute of Technology, Atlanta, Ga., 1990).